Tracer transport in the Unified Model

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The plan of attack

- The Unified Model
- Some notation and nomenclature
- The semi-Lagrangian scheme
- ENDGame
- Does it matter?
- SLICE – recovering conservation
- Conservation in LAMs
- GungHo!
- Bibliography
- Transport options in ROSE
THE UNIFIED MODEL
Unified Model

Brown et al. (2013)

- Operational forecasts
  - Mesoscale (resolution approx. 1.5km)
  - Global scale (resolution approx. 10km)

- Global and regional climate predictions
  - Resolution around 120km
  - Run for 10-100-... years

- Seasonal predictions
  - Resolution approx. 60km

- Research mode
  - Resolution 1km - 10m

> 25 years old
The consequence of unification

The same scheme has to continue to work

A factor of 1000 between these

300 km

30 km

300 m
Global model cf. other centres

* Parameters: PMSL, 500hPA GPH, 250hPa/850hPA Winds; Range: T+24 to T+120

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NOTATION AND NOMENCLATURE
Notation

- Let $\rho_X$ denote the *density, concentration, or mass per unit volume* of species $X$

- Let $\rho_d$ denote the density of *dry air*

- Then $m_X = \rho_X/\rho_d$ is the *mixing ratio* of species $X$

- By definition $m_d = 1$
Conservative form

Densities/concentrations transported according to:

\[
\frac{\partial \rho_x}{\partial t} + \nabla \cdot (U \rho_x) = 0 \quad \text{Eulerian flux form}
\]

\[
\frac{D}{Dt} \left( \int_V \rho_x \, dV \right) = 0 \quad \text{Lagrangian form (V=air parcel)}
\]
Advective form

Mixing ratios/parcel labels (e.g. age of air, mass of air parcel) are transported according to:

\[
\frac{\partial m_X}{\partial t} + \mathbf{U} \cdot \nabla m_X = 0 \quad \text{Eulerian form}
\]

\[
\frac{Dm_X}{Dt} = 0 \quad \text{Lagrangian form}
\]
THE SEMI-LAGRANGIAN SCHEME
From nature to a computer

- $\frac{DF}{Dt} = 0$ a natural form
- Integrate along the path a fluid parcel follows

\[ F(x + dx, t + dt) = F(x, t) \text{ where } \frac{dx}{dt} = U \]
Lagrangian & semi-Lagrangian

- **Lagrangian** model simply tracks air parcels
- This is the basis of the NAME model for plumes etc
- But, generally end up with very inhomogeneous distribution, requires interpolation/aggregation to where need answer
- **Semi-Lagrangian** schemes try to maintain the benefits of Lagrangian approach but on Eulerian grid
Semi-Lagrangian

- Arrival point, $X_A$, always a grid point
- Departure point, $X_D$, in general anywhere
- Two steps:
  - Evaluate trajectory, i.e. where $X_D$ is relative to $X_A$
  - Evaluate transported field at $X_D$

Staniforth and Côté (1991)
Benefits

- Excellent dispersion
  - Captures well the speed of propagation of waves
  - Key for good weather prediction
- Appropriate level of scale selective damping
- Excellent stability
  - Depends on physical (inverse) time scale \( \frac{dU}{dX} \), not numerical (inverse) time scale \( \frac{U}{\Delta X} \)
  - Particularly beneficial in large scale flows (cf. jets)
  - And in polar regions (operationally, polar \( \Delta X = 12 \) m, \( dt = 4 \) mins, and CFL = 1 for \( U = 5 \) cm/s!)
An example

Kohei Aranami (MetO/JMA)
Slotted cylinder test case

Initial Conditions

Kohei Aranami (MetO/JMA)
Disbenefits

- Lack of locality due to large time step, means departure point can be long way from arrival

- Conservation - consider cubic interpolation:

\[
\begin{align*}
  m_{-3}^n & \quad m_{-2}^n & \quad m_{-1}^n & \quad m_0^n & \quad m_{n+1}^n & \quad m_{n+2}^n & \quad m_{n+3}^n & \quad m_{i+1}^{n+1} & \quad m_{i+2}^{n+1} & \quad m_{i+3}^{n+1} \\
  x & \quad x & \quad x_D & \quad x & \quad x & \quad x_A \\
  x & \quad x & \quad x_D & \quad x & \quad x & \quad x_A \\
  x & \quad x & \quad x_D & \quad x & \quad x & \quad x_A \\
  x & \quad x & \quad x_D & \quad x & \quad x & \quad x_A \\
  x & \quad x & \quad x_D & \quad x & \quad x & \quad x_A
\end{align*}
\]
Conservation

- Even in case of interpolating mass (so don’t have to worry about density variations and non-uniform grid spacing), require:

\[
\sum_i m_i^n = \sum_i m_i^{n+1} = \sum_i \left( a_j m_j^{n(i-2)} + b_j m_j^{n(i-1)} + c_j m_j^{n(i)} + d_j m_j^{n(i+1)} \right)
\]

- For this to hold independent of mass distribution

\[
(a_{i+2} + b_{i+1} + c_i + d_{i-1}) m_i^n = m_i^n
\]

which is only true if wind is uniform

- \([\text{Cf. } a_i + b_i + c_i + d_i = 1]\)
ENDGame: Even Newer Dynamics for General atmospheric modelling of the environment
(Operational since 2014; Wood et al 2014)
Transport in ENDGame I

- Semi-Lagrangian scheme applied to all variables
- Special handling of vector aspects for wind
- Lagrangian interpolation:
  - Horizontal
    - Bi-cubic for all variables
  - Vertical
    - Cubic for wind components
    - Cubic-Hermite for potential temperature and moisture variables
    - Quintic for all other tracers
Transport in ENDGame II

- **Conservation:**
  - Priestley algorithm (optionally) applied to moisture and tracer variables *and* potential temperature

- **Monotonicity:**
  - Bermejo and Staniforth (optionally) applied to moisture and tracer variables *and* potential temperature
Dry mass conservation

- Without mass fixer relative change in total mass per time step is $O(10^{-5})$
- $\Rightarrow$ apply multiplicative fixer every time step
- Important that it preserves potential energy

- Achieved by:
  $$\rho^{n+1} = (A + Bz)\rho^*$$
  $$A$$ and $$B$$ chosen such that
  $$\sum \rho^{n+1} dV = \sum \rho^n dV$$
  $$\sum \rho^{n+1} gz dV = \sum \rho^* gz dV$$
Priestley algorithm

- Notes that loss of conservation arises from interpolation

- Compares low-order (specifically linear) interpolation with a high-order scheme (e.g. cubic or quintic)

- Argues that where these are different is where conservation will be lost

- Therefore adjusts high-order interpolated field proportionately to that difference

- Formally non-local but attempts to localize

Priestley (1993)
Monotonicity algorithm

- Higher-order interpolation scheme more accurate on smooth data
  - Cubic Lagrange is 3rd order accurate in space
- But applied to unsmooth data it will create overshoots and undershoots
- When this occurs high-order interpolation is not appropriate or sensible
- Could reduce the order progressively
- Pragmatic: limit the interpolated value to be bounded by the 8 values surrounding departure point

Bermejo and Staniforth (1992)
DOES IT MATTER WHAT WE DO?
Temperature bias in 20 year AMIP run

ENDGame zonal mean temperature

EG - ND

ND - ERA

EG - ERA

(ND=New Dynamics; EG=ENDGame; ERA=ERA-Interim)
\[ \frac{\partial \theta}{\partial t} \approx -w \frac{\partial \theta}{\partial z} \]
Impact of cubic Hermite + Priestley

Second-order centred → cubic Lagrange

ND bias → cubic Hermite → Priestley on potential temperature → EG bias

David Walters (Met Office)
SLICE:
SEMI-LAGRANGIAN INHERENTLY CONSERVATIVE AND EFFICIENT

RECOVERING CONSERVATION...
Conservative semi-Lagrangian

- Inherent conservation $\Rightarrow$ must use density or concentration, $\rho_X$

- But instead of usual Eulerian flux form

$$\frac{\partial \rho_X}{\partial t} + \nabla \cdot (U \rho_X) = 0$$

- Use Lagrangian form:

$$\frac{D}{Dt} \left( \int_V \rho_X \, dV \right) = 0$$
Conservative semi-Lagrangian

- Integrate along trajectory:

\[
\int_{V_A} \rho_X^{n+1} \, dV = \int_{V_D} \rho_X^n \, dV
\]

- Rearrange as:

\[
\rho_X^{n+1} = \frac{1}{V_A} \left( \int_{V_D} \rho_X^n \, dV \right)
\]

CONSERVATION IN LIMITED AREA MODELS...
LAM Conservation (budget)

- SL alone good
- Monotonicity messes this up
- Conservation recovers accuracy
- And gives exact budget

PLF: Aranami, Davies and Wood (2014)
ZLF: Zerroukat & Shipway (2017)
GUNGHOO INTO THE FUTURE!
Scalability

$T_{7344}/T_N$

Perfect scaling

ENDGame

6.5 km global

$7344$ cores

$1$ node $= 36$ cores

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Paul Selwood & Andy Malcolm (Met Office)
The finger of blame...

- At 17km resolution, grid spacing near poles = 35m
- At 10km spacing = 12m
- At 1km reduces to 12 cm!
GungHo!

Globally
Uniform
Next
Generation
Highly
Optimized

NERC SCIENCE OF THE ENVIRONMENT
Science & Technology Facilities Council

“Working together harmoniously”
Where are we?

- Cubed-sphere is principal contender
- But grid non-orthogonal
- To maintain same accuracy using mixed finite-element spatial discretization...
- ...coupled with an *Eulerian flux form* transport scheme (either finite element or finite volume)
- Redesigning Unified Model
  - F2003
  - Separation of concerns - PSyKAI
- Targeting mid-2020’s
Straka cold bubble

Low Order Mixed FEM; GH = 50 m; EG = 50 m
Orographic gravity waves

Low-order Mixed FEM; stratified flow over a small hill; uniform Cartesian domain

GungHo

ENDGame
Baroclinic wave

Low Order Mixed FEM; GH = C96 ~ 1 degree; EG = 1 degree

Day 8 surface pressure

GungHo

ENDGame
Thank you!

Questions?

See extra slides for Bibliography and How to select options in UM
Bibliography


TRACER TRANSPORT OPTIONS IN ROSE

with thanks to Chris Smith
Interpolation options in Rose:

vn ≥ 10.6

Separate options for moisture and tracers...

... with range of interpolation schemes...
Interpolation options in Rose: $vn \geq 10.6$

... and new options for monotonicity
Conservation options in Rose: $vn \geq 10.1$

Tracer conservation now has the option to use the Priestley (1993) algorithm: