

Tracer transport in the Unified Model

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The plan of attack!

- The Unified Model
- Some notation and nomenclature
- The semi-Lagrangian scheme
- New Dynamics
- ENDGame
- Does it matter?
- SLICE recovering conservation
- Conservation in LAMs
- GungHo!
- Bibliography
- Transport options in the UMUI & ROSE



Brown et al. (2013)

Operational forecasts

Mesoscale (resolution approx. 4km, 1.5km)

➢Global scale (resolution approx. 17km)

 Global and regional climate predictions

- Resolution around 120km
- ➢ Run for 10-100-... years

Seasonal predictions

➢ Resolution approx. 60km

Research mode

Resolution 1km - 10m





The consequence of unification

Met Office 300 km





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Notation and nomenclature



 Let p_X denote the *density*, *concentration*, or *mass per unit volume* of species X

- Let ρ_d denote the density of *dry air*

• Then $m_X = \rho_X / \rho_d$ is the *mixing ratio* of species X

By definition m_d = 1



Densities/concentrations transported according to:

$$\frac{\partial \rho_{X}}{\partial t} + \nabla \cdot (U \rho_{X}) = 0 \qquad \begin{array}{c} \text{Eulerian flux} \\ \text{form} \end{array}$$
$$\frac{D}{Dt} \left(\int_{V} \rho_{X} dV \right) = 0 \qquad \begin{array}{c} \text{Lagrangian form} \\ \text{(V=air parcel)} \end{array}$$



Mixing ratios/parcel labels (e.g. age of air, mass of air parcel) are transported according to:

$$\frac{\partial m_{X}}{\partial t} + \boldsymbol{U} \cdot \nabla m_{X} = 0 \quad \text{Eulerian form}$$

$$\frac{Dm_{X}}{Dt} = 0$$
 Lagrangian form



The semi-Lagrangian scheme



- DF/Dt=0 a natural form
- Integrate along the path a fluid parcel follows



• $F(\mathbf{x}+d\mathbf{x},t+dt) = F(\mathbf{x},t)$ where $d\mathbf{x}/dt=\mathbf{U}$



Lagrangian & semi-Lagrangian

- Lagrangian model simply tracks air parcels
- This is the basis of the NAME model for plumes etc
- But, generally end up with very inhomogeneous distribution, requires interpolation/aggregation to where need answer
- Semi-Lagrangian schemes try to maintain the benefits of Lagrangian approach but on Eulerian grid



- Arrival point, X_A, always a grid point
- Departure point, X_D, in general anywhere
- Two steps:

 \succ Evaluate trajectory, i.e. where X_D is relative to X_A

 \succ Evaluate transported field at X_D

Staniforth and Côté (1991)



Excellent dispersion

Captures well the speed of propagation of waves
Key for good weather prediction

- Appropriate level of scale selective damping
- Excellent stability

➢Depends on physical (inverse) time scale dU/dX, not numerical (inverse) time scale U/∆X

➢Particularly beneficial in large scale flows (cf. jets)

And in polar regions (operationally, polar $\Delta X=35$ m, dt = 7.5 mins, and CFL = 1 for U=8 **cm**/s!)





Kohei Aranami (MetO/JMA)



Initial Conditions





0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Kohei Aranami (MetO/JMA)



- Lack of locality due to large time step, means departure point can be long way from arrival
- Conservation consider cubic interpolation:





 Even in case of interpolating mass (so don't have to worry about density variations and nonuniform grid spacing), require:

$$\sum_{i} \mathbf{m}_{i}^{n} = \sum_{i} \mathbf{m}_{i}^{n+1} = \sum_{i} \left(\mathbf{a}_{j} \mathbf{m}_{j(i)-2}^{n} + \mathbf{b}_{j} \mathbf{m}_{j(i)-1}^{n} + \mathbf{c}_{j} \mathbf{m}_{j(i)}^{n} + \mathbf{d}_{j} \mathbf{m}_{j(i)+1}^{n} \right)$$

For this to hold independent of mass distribution

$$(a_{i+2} + b_{i+1} + c_i + d_{i-1})m_i^n = m_i^n$$

which is only true if wind is uniform

• [Cf.
$$a_i + b_i + c_i + d_i = 1$$
]



New Dynamics



Transport in New Dynamics I

Semi-Lagrangian scheme applied to:

≻All moisture variables and all tracers

>Wind components (special handling of vector aspects)

Horizontal aspects of potential temperature advection

>Nearest grid point in vertical for potential temperature

- Eulerian flux scheme used for dry density
- And Eulerian advection scheme for residual vertical advection of potential temperature



Transport in New Dynamics II

Lagrangian interpolation:

Bi-(quasi-)cubic in horizontal and quintic in vertical for moisture variables and all tracers

Tri-(quasi-)cubic for wind components

Bi-(quasi-)cubic for horizontal aspects of potential temperature advection

Conservation:

Priestley algorithm (optionally) applied to moisture and tracer variables

Monotonicity:

Bermejo and Staniforth (optionally) applied to moisture and tracer variables



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- Notes that loss of conservation arises from interpolation
- Compares low-order (specifically linear) interpolation with a high-order scheme (e.g. Cubic or quintic)
- Argues that where these are different is where conservation will be lost
- Therefore adjusts high-order interpolated field proportionately to that difference
- Formally non-local but attempts to localize





Monotonicity algorithm

 Higher-order interpolation scheme more accurate on smooth data

Cubic Lagrange is 3rd order accurate in space

- But applied to unsmooth data it will create overshoots and undershoots
- When this occurs high-order interpolation is not appropriate or sensible
- Could reduce the order progressively
- Pragmatic: limit the interpolated value to be bounded by the 8 values surrounding departure point



ENDGame: Even Newer Dynamics for General atmospheric modelling of the environment



- Motivation: New Dynamics numerically unstable
 - Mixed semi-Lagrangian/Eulerian approach unstable
 - >Eulerian approach probably root cause of polar noise
 - More diffusion needed to control instabilities impact on accuracy and scalability
 - Extrapolated winds used in departure point evaluation – unstable
 - "Non-interpolating in vertical" for potential temperature – cause of valley cooling
- ENDGame

>Adopts iterated approach akin to Canadian GEM model

➢Model significantly more: stable, scalable and accurate



Semi-Lagrangian scheme applied to:

All variables!

Transport in ENDGame II

Met Office

- Lagrangian interpolation:
 - Horizontal
 - Bi-cubic for all variables
 - Vertical
 - Cubic for wind components
 - Cubic-Hermite for potential temperature and moisture variables
 - Quintic for all other tracers
- Conservation:
 - Priestley algorithm (optionally) applied to moisture and tracer variables and potential temperature
- Monotonicity:
 - Bermejo and Staniforth (optionally) applied to moisture and tracer variables and potential temperature



Met Office

- Without mass fixer relative change in total mass per time step is O(10⁻⁵)
- \Rightarrow apply multiplicative fixer every time step
- Important that it preserves potential energy
- Achieved by:

$$\rho^{n+1} = (A + Bz) \rho^*$$

• A and B chosen such that

$$\sum \rho^{n+1} dV = \sum \rho^n dV$$

$$\sum \rho^{n+1} gz dV = \sum \rho^* gz dV$$



Does it matter what we do?







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Chris Smith (Met Office)



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David Walters (Met Office)



SLICE: Semi-Lagrangian Inherently Conservative and Efficient

Recovering conservation...



Conservative semi-Lagrangian

- Inherent conservation \Rightarrow must use density or concentration, ρ_X
- But instead of usual Eulerian flux form

$$\frac{\partial \rho_{X}}{\partial t} + \nabla \cdot \left(\boldsymbol{U} \rho_{X} \right) = 0$$

Use Lagrangian form:

$$\frac{D}{Dt} \left(\int_{V} \rho_{X} dV \right) = 0$$

Zerroukat, Wood & Staniforth (2002)



Integrate along trajectory:





1. Cascade 3D reconstruction into 3 x 1D reconstructions:

2. Perform 1D reconstructions

 $\Lambda_{i-1/2}$ $|\Lambda_{i+1/2}|$ Λ_{i} ILCV_{i+1,j+2} $\Phi_{j+1/2}$ Φi ¢j+1/2 $\Phi_{\mathsf{j}\text{-}\mathsf{1/2}}$ LCV _{i,j} ¢j-1/2 ECV _{i, j} IECV_{i,j-1}

 $\lambda_{i+1/2}$

λi















Conservation in Limited Area Models...

LAM Conservation (budget)

Met Office

- SL alone good
- Monotonicity messes this up
- Conservation recovers accuracy
- And gives exact budget





Met Office

- Noting that monotonicity has greatest impact:
 - 1. Switch off monotonicity
 - 2. Use quintic Lagrange interpolation in *all* directions
 - 3. Tidy any negative condensed moisture variables by absorbing negative values directly into local vapour field
 - 4. Adjust potential temperature for phase change
- Could be used together with LAM conservation



GungHo into the future!





Met Office

- At 25km resolution, grid spacing near poles = 75m
- At 17km resolution, grid spacing near poles = 35m
- At 10km reduces to 12m!







- Cubed-sphere is principal contender
- But grid non-orthogonal
- To maintain same accuracy using mixed finiteelement spatial discretization...
- ...coupled with an *Eulerian flux form* transport scheme (either finite element or finite volume)
- Redesigning Unified Model

≻F2003

Separation of concerns - PSyKAI

Targeting early 2020's



Thank you! Questions?

See extra slides for Bibliography and How to select options in UM



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Tracer transport options in UMUI and Rose with thanks to Chris Smith

$\underset{Met Office}{\text{Met Office}} \text{ Interpolation options in UMUI:}$

Moisture and tracers treated in the same

High Order Scheme: 0 = tri-linear >0 = bi-cubic in horizontal ...



Met Office $Vn \le 8.6$ Met Office $Vn \le 8.6$

1 = monotone clipping 0 = non-monotone (if high order)



$\underset{Met Office}{\ref{Conservation options in}} Conservation options in$



$\underset{Met Office}{\texttt{Met Office}} \text{ Tracer conservation in UMUI:}$

Just two clicks in the Section 11 panel:





Now separate options for moisture and tracers:

M Advection 🗙		
monotone_sche monotone scheme density and tracers	eme e for (theta, moisture, winds, s)	1 1 0 0 1 < >
high_order_sche high order scheme density and tracers	eme :s for (theta, moisture, wind, s)	87117<>

$\underset{Met Office}{\text{Met Office}} \text{ Interpolation options in Rose:}$

	VM Advection 🔀		
monotone_scheme			
	monotone scheme for (theta 🚸 density and tracers)		Help for namelist:run_sl=high_order_scheme ×
	high_order_scheme high order schemes for (theta density and tracers)	High order schemes: 0-Linear interpolation - no high order scheme 1-Cubic Lagrange interpolation 2-Quintic Lagrange interpolation	
And lots of useful help:		2-Quintic Lagrange interpolation 3-ECMWF quasi-cubic interpolation (New dynamics only) 4-ECMWF monotone quasi-cubic interpolation (New dynamics only) 5-Bi-cubic Lagrange interpolation in the horizontal, linear interpolation in the vertical 6-ECMWF quasi-cubic interpolation in the horizontal, quintic in the vertical (New dynamics only) 7-cubic Lagrange interpolation in the horizontal, quintic in the vertical 8-LOCH: bi-cubic Lagrange in the horizontal; C1-Hermite cubic with quadratic derivative estimates in the vertical (ENDGame only) 9-HOCH: bi-cubic Lagrange in the horizontal; C1-Hermite cubic with quartic derivative estimates in the vertical (ENDGame only)	
			Close 💥

$\underset{Met Office}{\ref{eq:MetOffice}} Conservation options in Rose:$

Tracer conservation now has the option to use the Priestley (1993) algorithm:

