

# Tracer transport in the Unified Model

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## The plan of attack!

- The Unified Model
- Some notation and nomenclature
- The semi-Lagrangian scheme
- ENDGame
- Does it matter?
- SLICE – recovering conservation
- Conservation in LAMs
- GungHo!
- Bibliography
- Transport options in ROSE



# Unified Model

*Brown et al. (2013)*

- Operational forecasts

- Mesoscale (resolution approx. 1.5km)
- Global scale (resolution approx. 17km)

- Global and regional climate predictions

- Resolution around 120km
- Run for 10-100-... years

- Seasonal predictions

- Resolution approx. 60km

- Research mode

- Resolution 1km - 10m

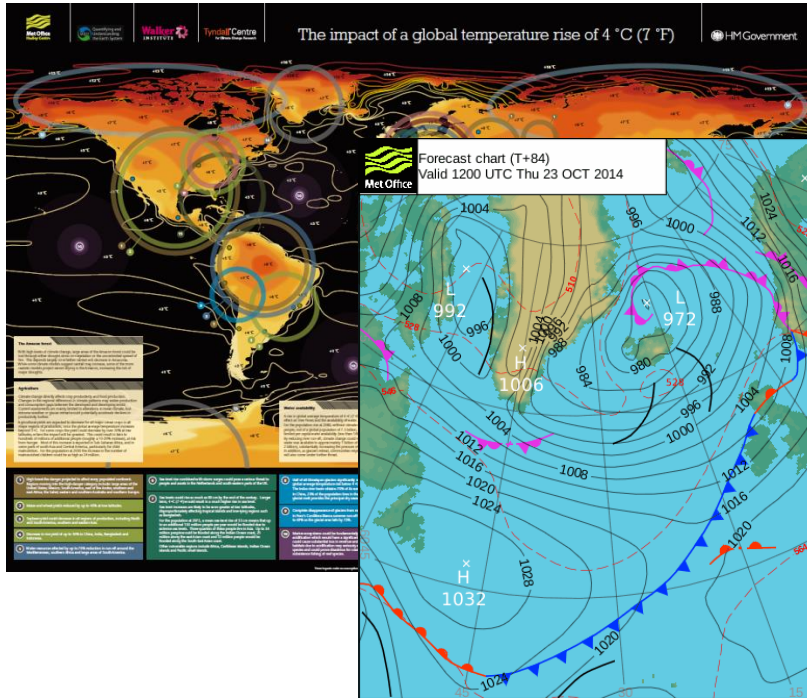
> 25 years old



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# The consequence of unification

300 km

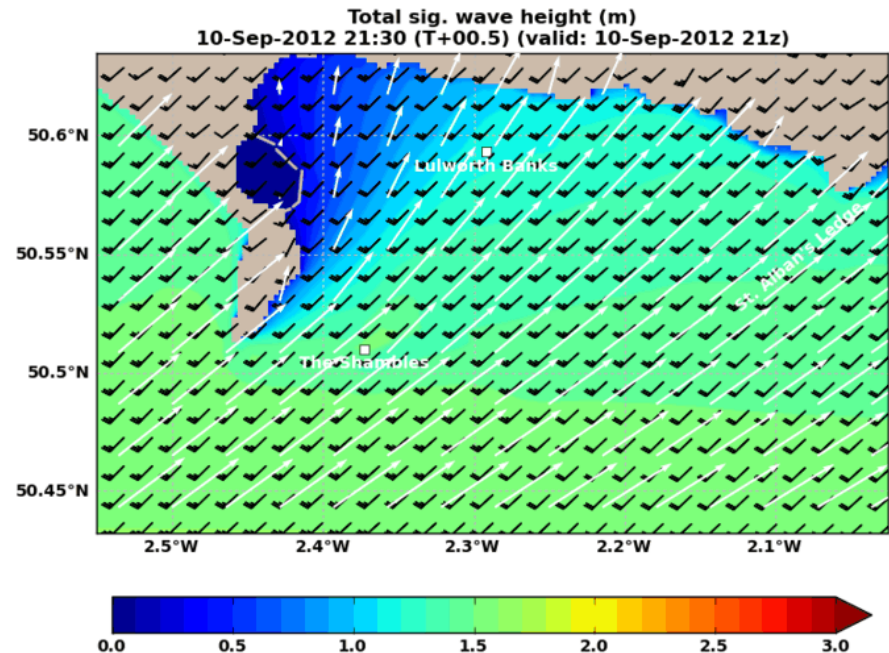


30 km

A factor of 1000  
between these

300 m

...the same scheme has  
to continue to work

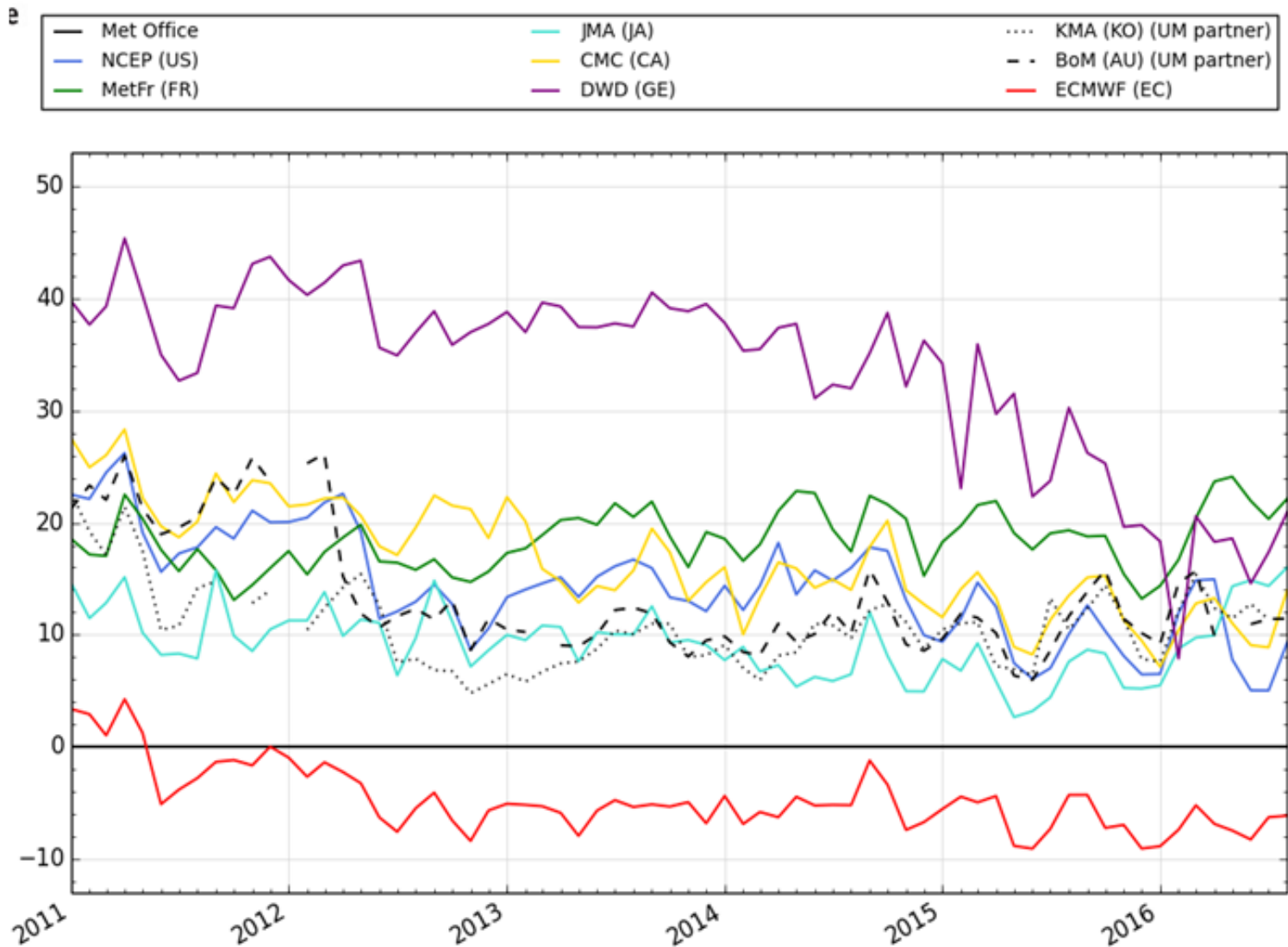




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# Global model cf. other centres

Skill\* Difference (%) relative to Met Office





# Notation and nomenclature

# Notation

- Let  $\rho_X$  denote the ***density, concentration, or mass per unit volume*** of species X
- Let  $\rho_d$  denote the density of ***dry air***
- Then  $m_X = \rho_X/\rho_d$  is the ***mixing ratio*** of species X
- By definition  $m_d = 1$

# Conservative form

Densities/concentrations transported according to:

$$\frac{\partial \rho_X}{\partial t} + \nabla \cdot (\mathbf{U} \rho_X) = 0 \quad \text{Eulerian flux form}$$

$$\frac{D}{Dt} \left( \int_V \rho_X dV \right) = 0 \quad \text{Lagrangian form (V=air parcel)}$$



# Advective form

Mixing ratios/parcel labels (e.g. age of air, mass of air parcel) are transported according to:

$$\frac{\partial m_X}{\partial t} + \mathbf{U} \cdot \nabla m_X = 0 \quad \text{Eulerian form}$$

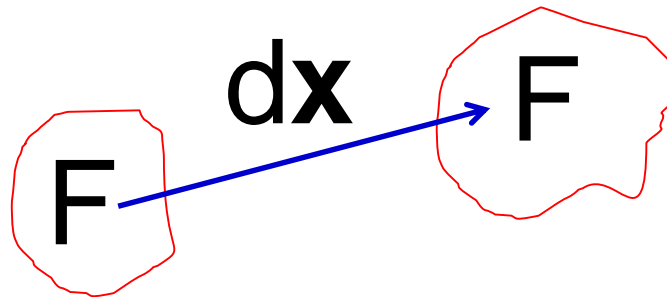
$$\frac{Dm_X}{Dt} = 0 \quad \text{Lagrangian form}$$



# The semi-Lagrangian scheme

# From nature to a computer

- $DF/Dt=0$  a natural form
- Integrate along the path a fluid parcel follows

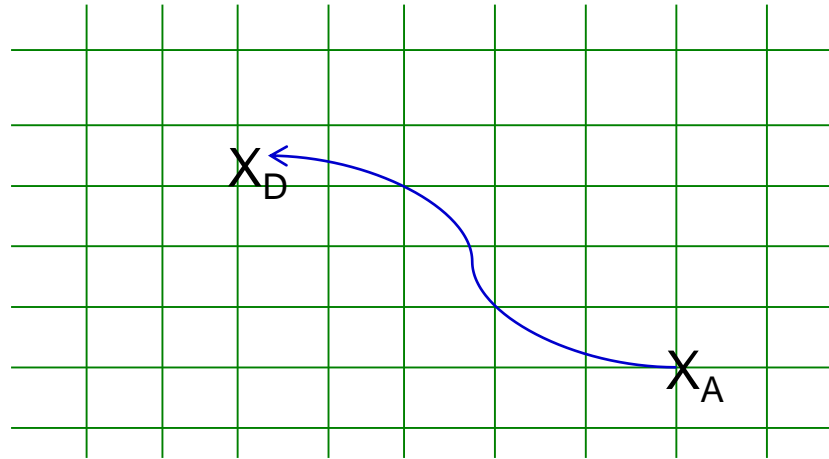


- $F(\mathbf{x}+d\mathbf{x},t+dt) = F(\mathbf{x},t)$  where  $d\mathbf{x}/dt=\mathbf{U}$

# Lagrangian & semi-Lagrangian

- **Lagrangian** model simply tracks air parcels
- This is the basis of the NAME model for plumes etc
- But, generally end up with very inhomogeneous distribution, requires interpolation/aggregation to where need answer
- **Semi-Lagrangian** schemes try to maintain the benefits of Lagrangian approach but on Eulerian grid

# Semi-Lagrangian



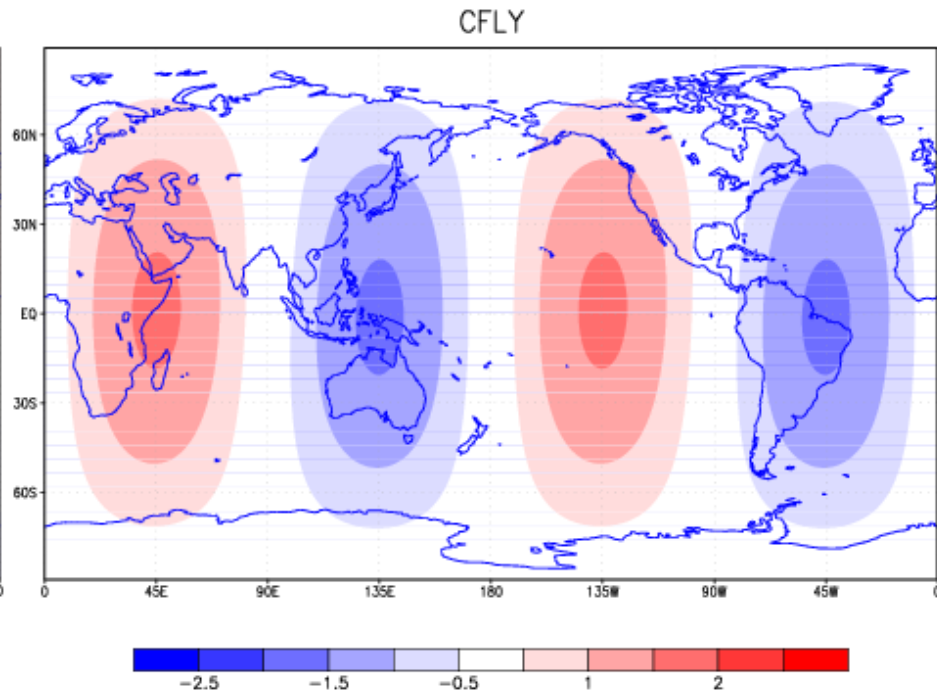
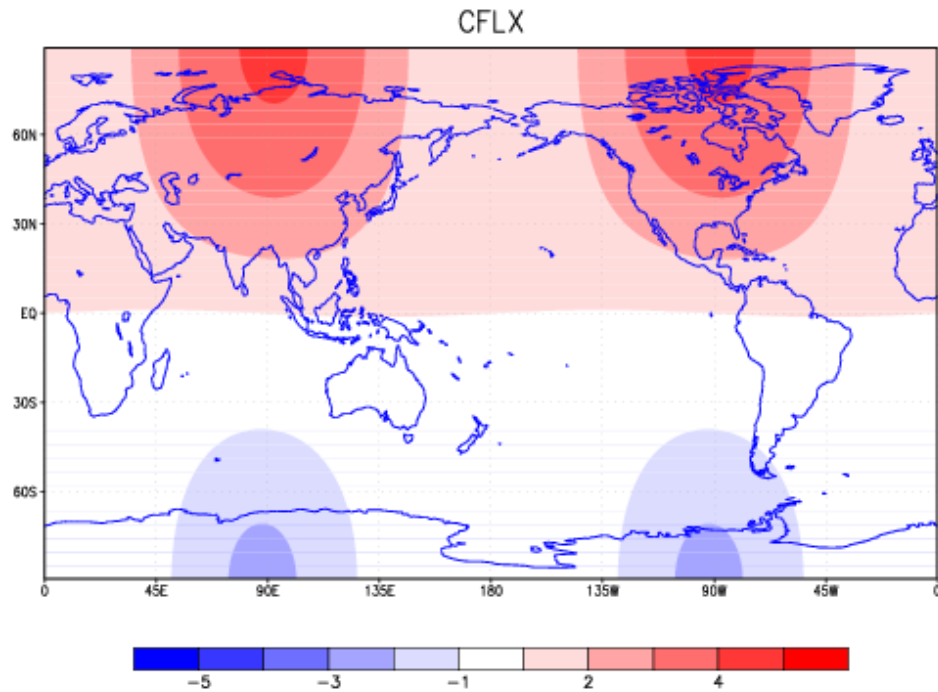
- Arrival point,  $X_A$ , always a grid point
- Departure point,  $X_D$ , in general anywhere
- Two steps:
  - Evaluate trajectory, i.e. where  $X_D$  is relative to  $X_A$
  - Evaluate transported field at  $X_D$

Staniforth and Côté (1991)

# Benefits

- Excellent **dispersion**
  - Captures well the speed of propagation of waves
  - Key for good weather prediction
- Appropriate level of **scale selective damping**
- Excellent **stability**
  - Depends on physical (inverse) time scale  $d\mathbf{U}/d\mathbf{X}$ , not numerical (inverse) time scale  $\mathbf{U}/\Delta\mathbf{X}$
  - Particularly beneficial in large scale flows (cf. jets)
  - And in polar regions (operationally, polar  $\Delta X=35$  m,  $dt = 7.5$  mins, and  $CFL = 1$  for  $U=8$  **cm/s**!)

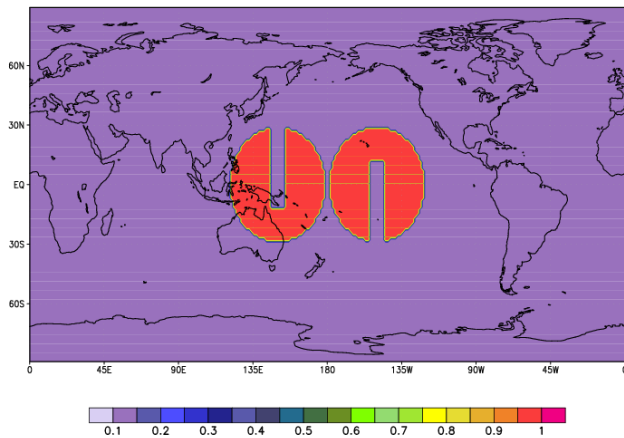
# An example



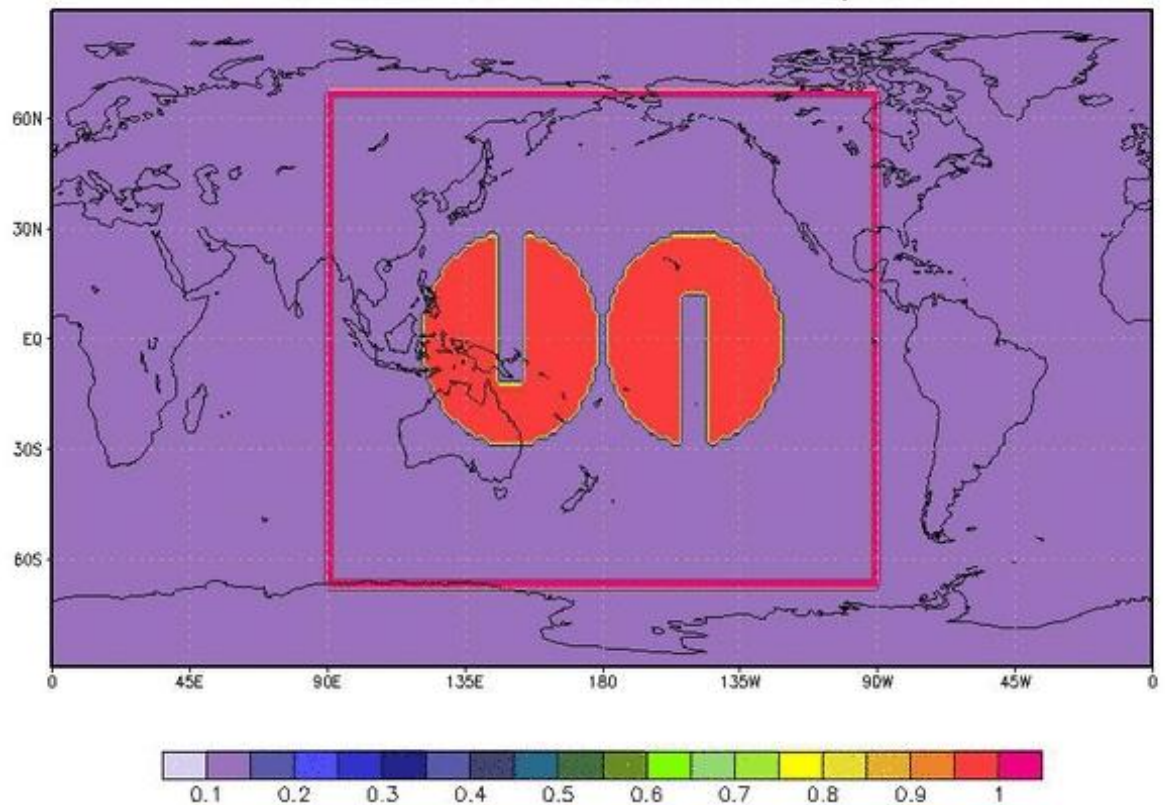
Kohei Aranami (MetO/JMA)

# Slotted cylinder test case

## Initial Conditions



SL-QMSL PLF : Fields at t=001/241

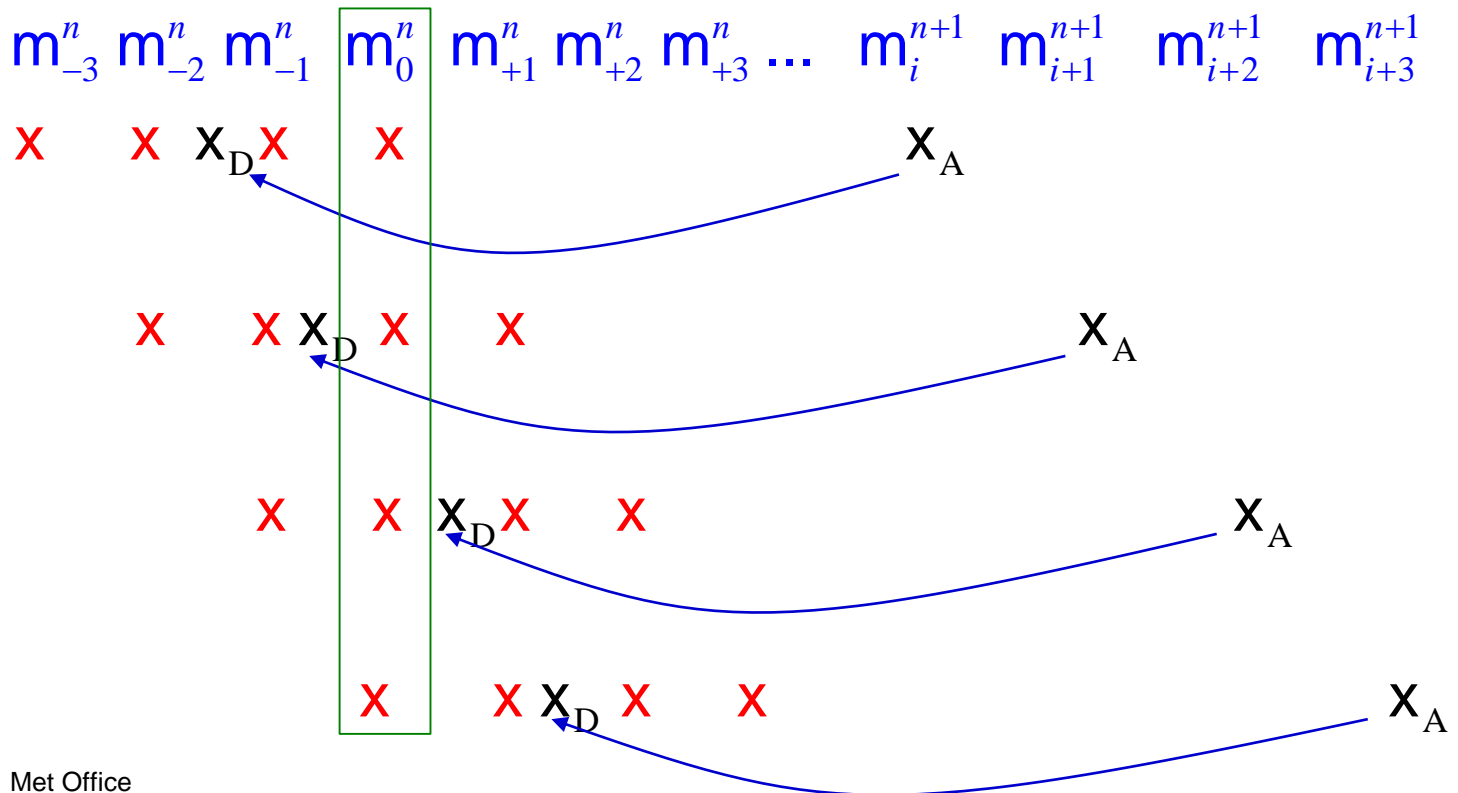


Kohei Aranami (MetO/JMA)



# Disbenefits

- Lack of locality due to large time step, means departure point can be long way from arrival
- Conservation - consider cubic interpolation:



# Conservation

- Even in case of interpolating mass (so don't have to worry about density variations and non-uniform grid spacing), require:

$$\sum_i m_i^n = \sum_i m_i^{n+1} = \sum_i \left( a_j m_{j(i)-2}^n + b_j m_{j(i)-1}^n + c_j m_{j(i)}^n + d_j m_{j(i)+1}^n \right)$$

- For this to hold independent of mass distribution

$$\left( a_{i+2} + b_{i+1} + c_i + d_{i-1} \right) m_i^n = m_i^n$$

which is only true if wind is uniform

- [Cf.  $a_i + b_i + c_i + d_i = 1$  ]



# ENDGame: Even Newer Dynamics for General atmospheric modelling of the environment

(Operational since 2012; Wood et al 2014)

# Transport in ENDGame I

- Semi-Lagrangian scheme applied to all variables
- Special handling of vector aspects for wind
- Lagrangian interpolation:
  - Horizontal
    - Bi-cubic for all variables
  - Vertical
    - Cubic for wind components
    - Cubic-Hermite for potential temperature and moisture variables
    - Quintic for all other tracers

# Transport in ENDGame II

- Conservation:
  - Priestley algorithm (optionally) applied to moisture and tracer variables **and** potential temperature
- Monotonicity:
  - Bermejo and Staniforth (optionally) applied to moisture and tracer variables **and** potential temperature

# Dry mass conservation

- Without mass fixer relative change in total mass per time step is  $O(10^{-5})$
- $\Rightarrow$  apply multiplicative fixer every time step
- Important that it preserves potential energy

- Achieved by:

$$\rho^{n+1} = (A + Bz) \rho^*$$

- $A$  and  $B$  chosen such that

$$\sum \rho^{n+1} dV = \sum \rho^n dV$$

$$\sum \rho^{n+1} gz dV = \sum \rho^* gz dV$$

# Priestley algorithm

- Notes that loss of conservation arises from interpolation
- Compares low-order (specifically linear) interpolation with a high-order scheme (e.g. Cubic or quintic)
- Argues that where these are different is where conservation will be lost
- Therefore adjusts high-order interpolated field proportionately to that difference
- Formally non-local but attempts to localize

Priestley (1993)

# Monotonicity algorithm

- Higher-order interpolation scheme more accurate on smooth data
  - Cubic Lagrange is 3<sup>rd</sup> order accurate in space
- But applied to unsmooth data it will create overshoots and undershoots
- When this occurs high-order interpolation is not appropriate or sensible
- Could reduce the order progressively
- Pragmatic: limit the interpolated value to be bounded by the 8 values surrounding departure point

Bermejo and Staniforth (1992)

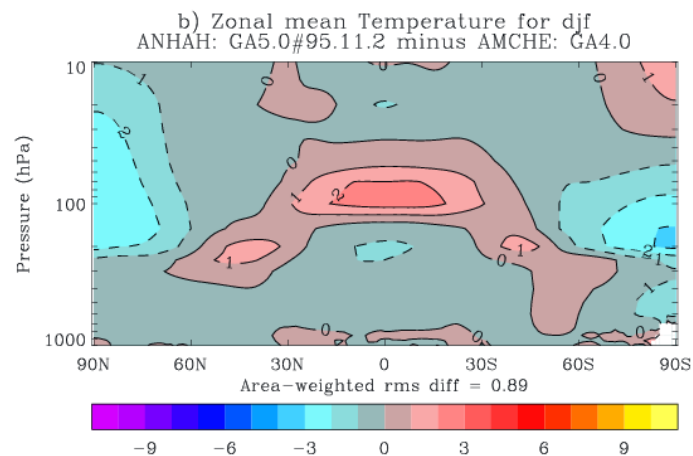
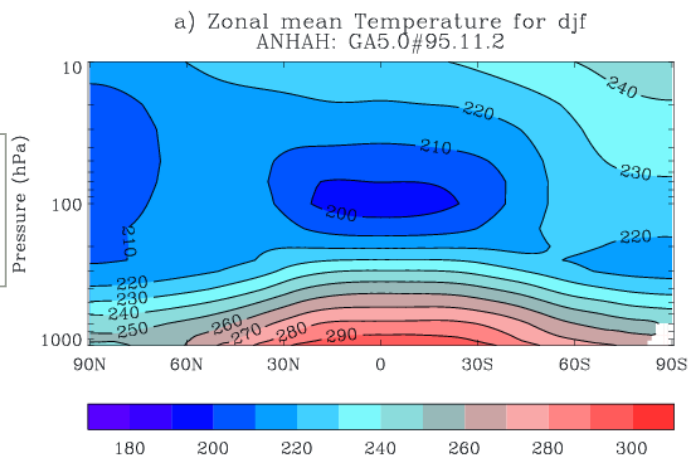




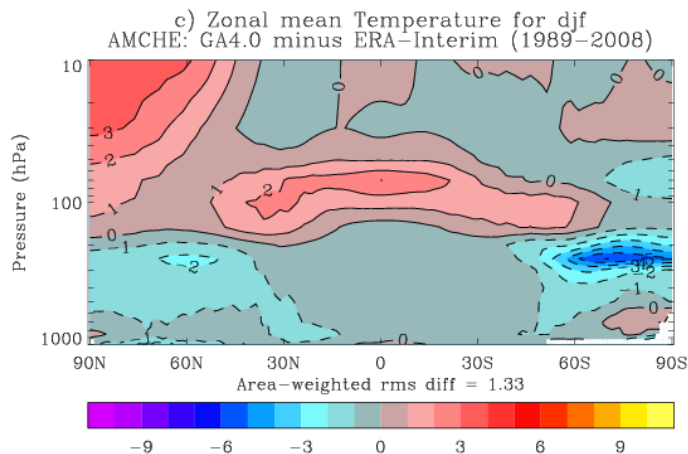
Does it matter what we do?

# Temperature bias in 20 year AMIP run

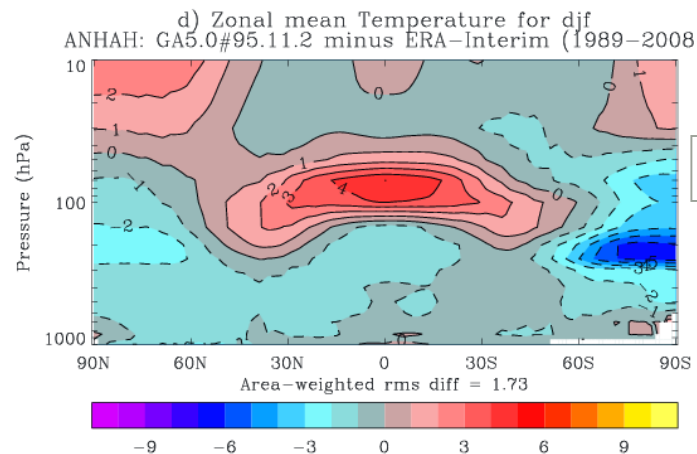
ENDGame  
zonal mean  
temperature



EG - ND



ND - ERA



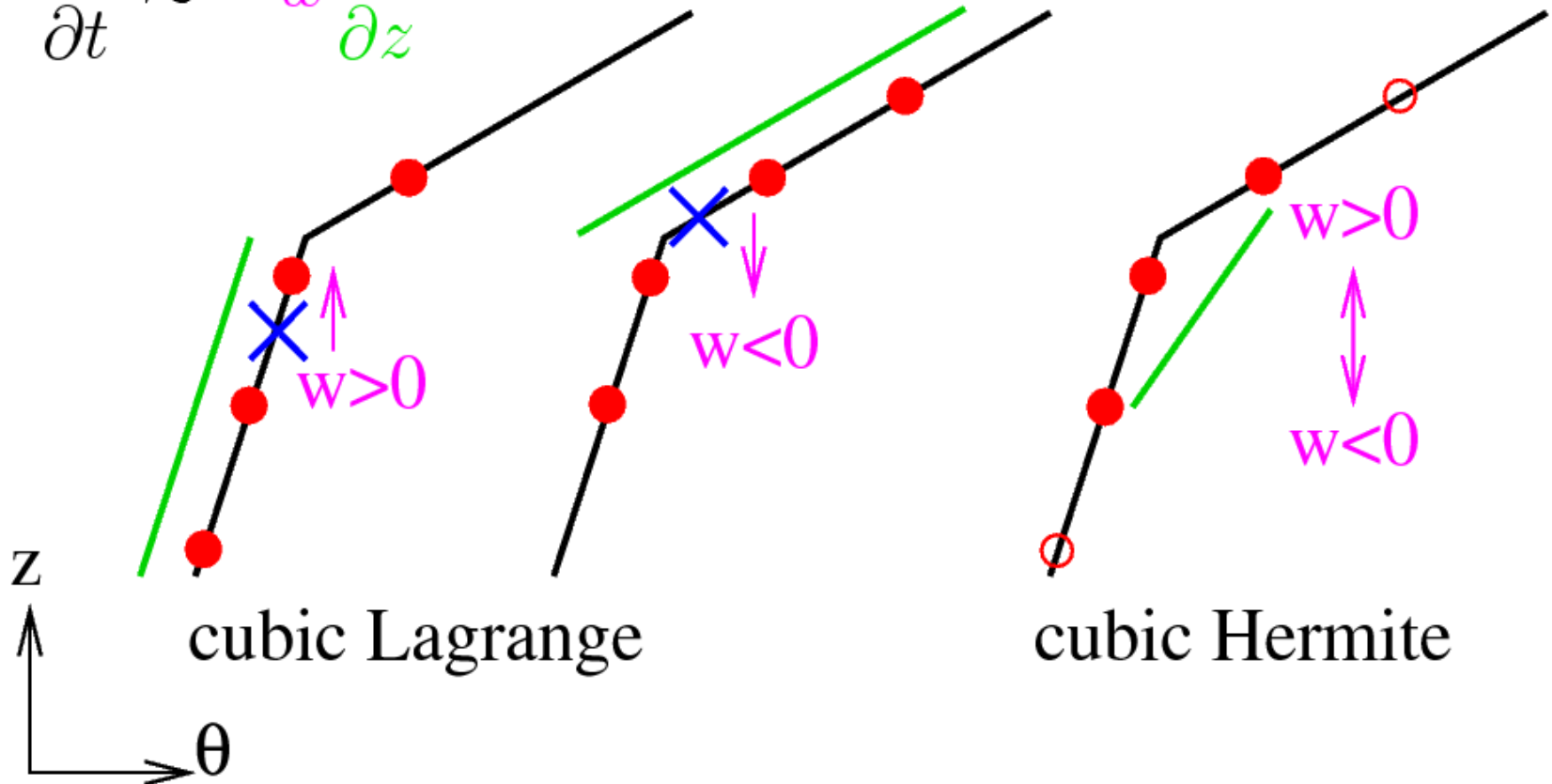
EG - ERA



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# Why?

$$\frac{\partial \theta}{\partial t} \approx -w \frac{\partial \theta}{\partial z}$$





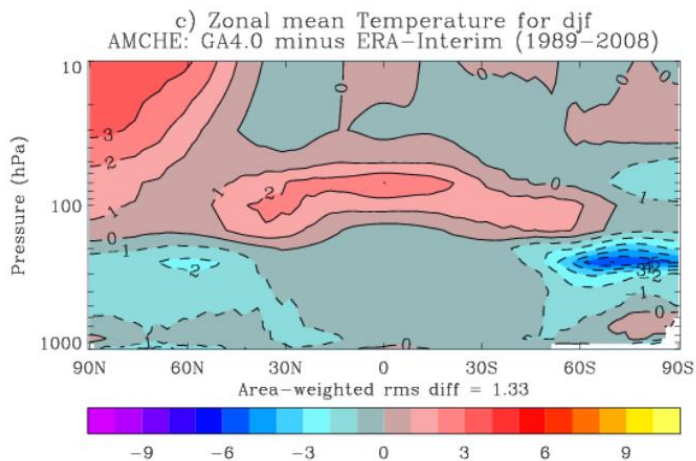
Met Office

# Impact of cubic Hermite + Priestley

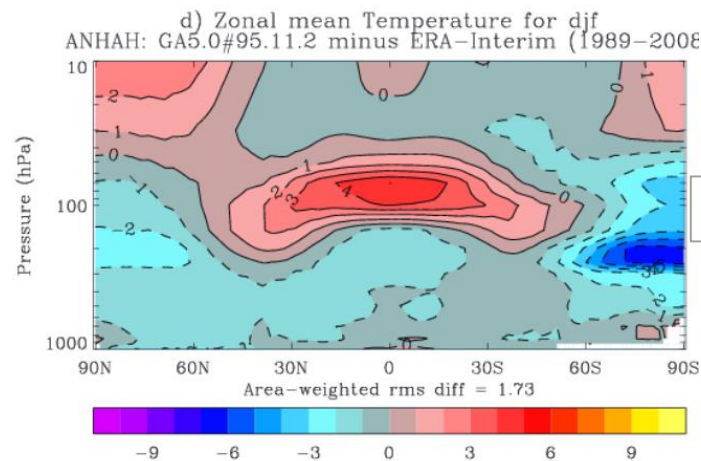
Second-order centred

cubic Lagrange

ND bias

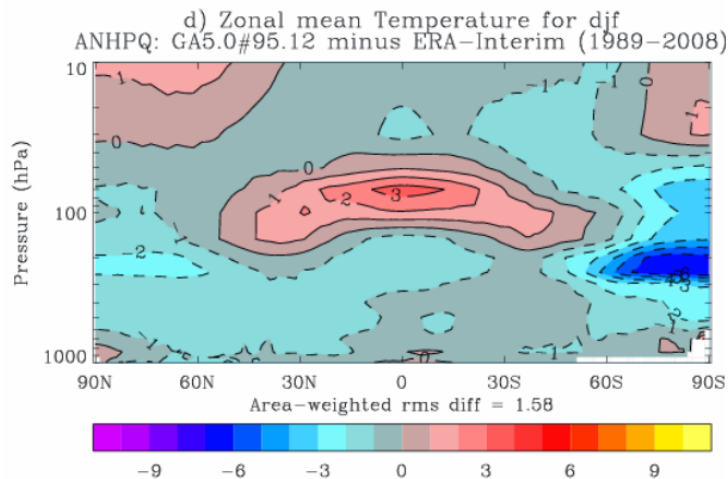
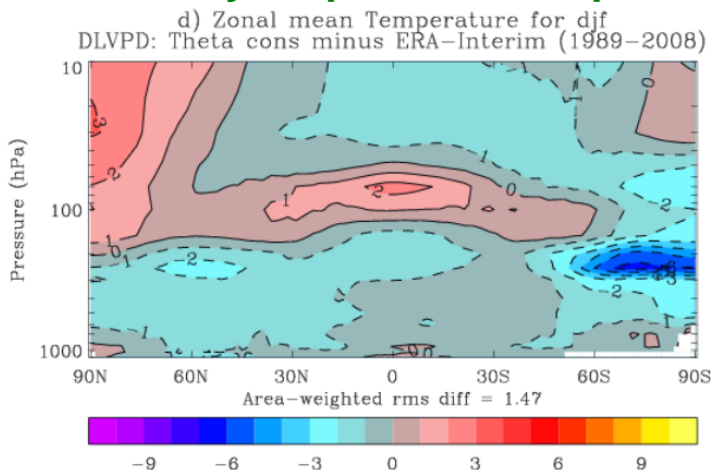


EG bias



Priestley on potential temperature

cubic Hermite





SLICE:  
Semi-Lagrangian Inherently Conservative and Efficient

Recovering conservation...

# Conservative semi-Lagrangian

- Inherent conservation  $\Rightarrow$  must use density or concentration,  $\rho_X$
- But instead of usual Eulerian flux form

$$\frac{\partial \rho_X}{\partial t} + \nabla \cdot (\mathbf{U} \rho_X) = 0$$

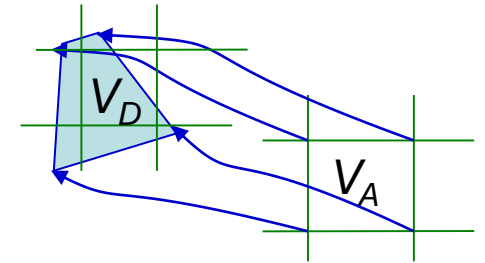
- Use Lagrangian form:

$$\frac{D}{Dt} \left( \int_V \rho_X dV \right) = 0$$

# Conservative semi-Lagrangian

- Integrate along trajectory:

$$\int_{V_A} \rho_X^{n+1} dV = \int_{V_D} \rho_X^n dV$$



- Rearrange as:

$$\rho_X^{n+1} = \frac{1}{V_A} \left( \int_{V_D} \rho_X^n dV \right)$$

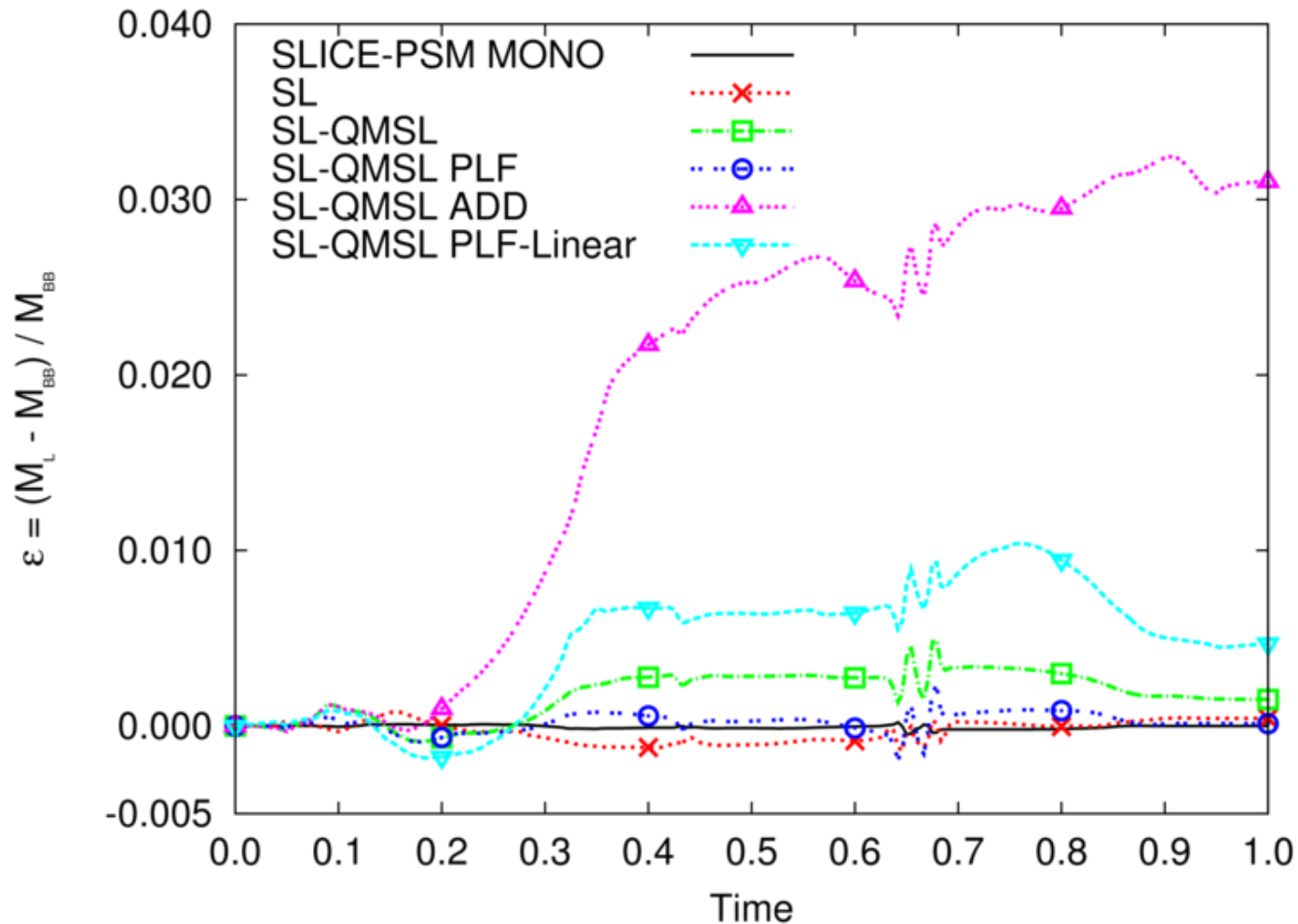


# Conservation in Limited Area Models...



# LAM Conservation (budget)

- SL alone good
- Monotonicity messes this up
- Conservation recovers accuracy
- And gives exact budget



PLF: Aranami, Davies and Wood (2014)

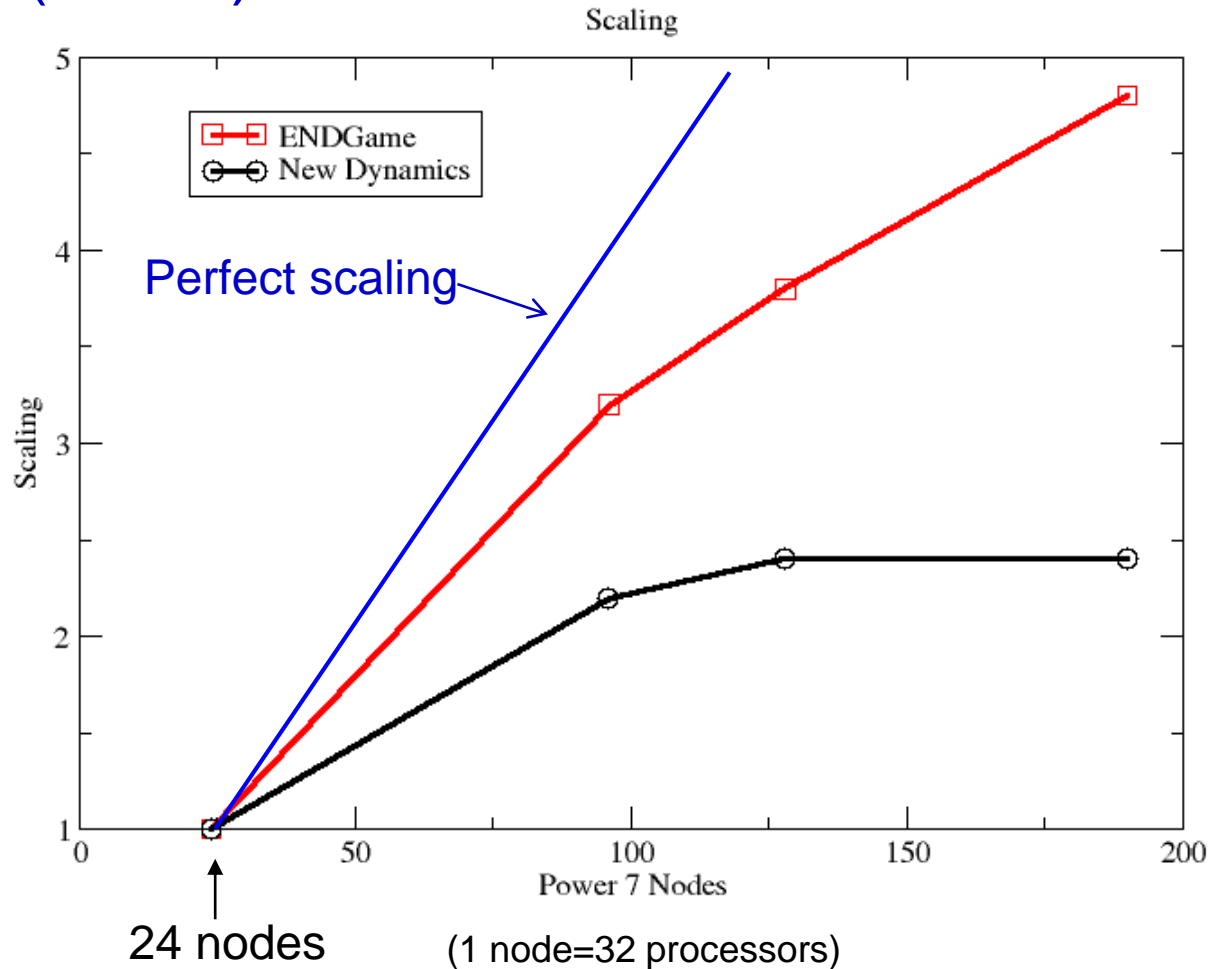
ZLF: Zerroukat & Shipway (2017)



GungHo into the future!

# Scalability

(17km) N768 - New Dynamics vs ENDGame



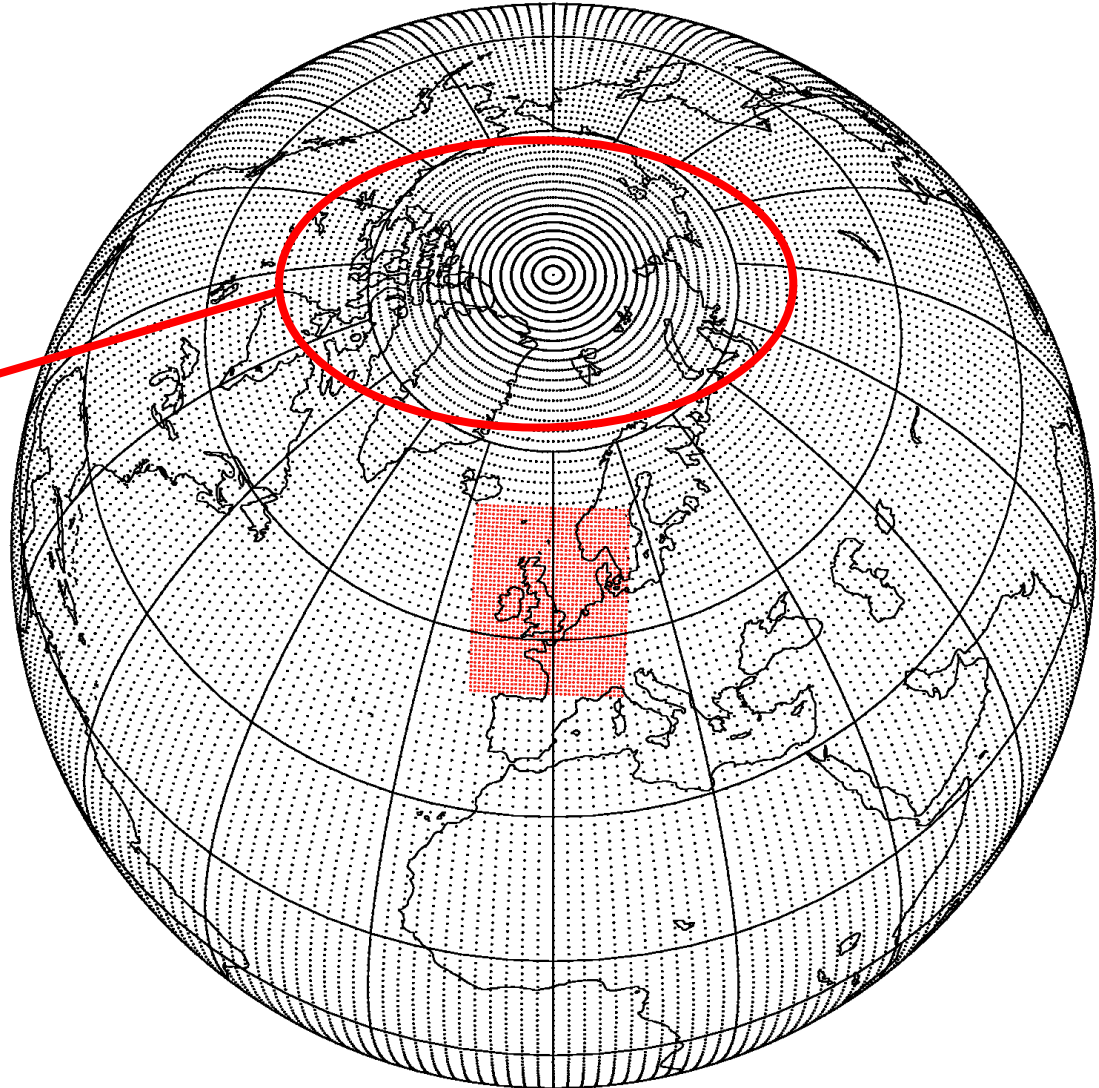
Andy Malcolm (Met Office)



Met Office

# The finger of blame...

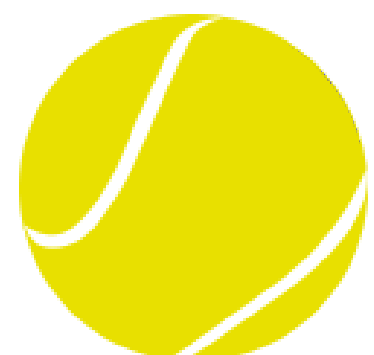
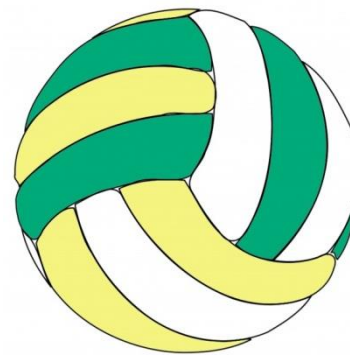
- At 25km resolution, grid spacing near poles = 75m
- At 17km resolution, grid spacing near poles = 35m
- At 10km resolution reduces to 12m!





# GungHo!

**Globally**  
**Uniform**  
**Next**  
**Generation**  
**Highly**  
**Optimized**



**Science & Technology**  
Facilities Council

**“Working together harmoniously”**

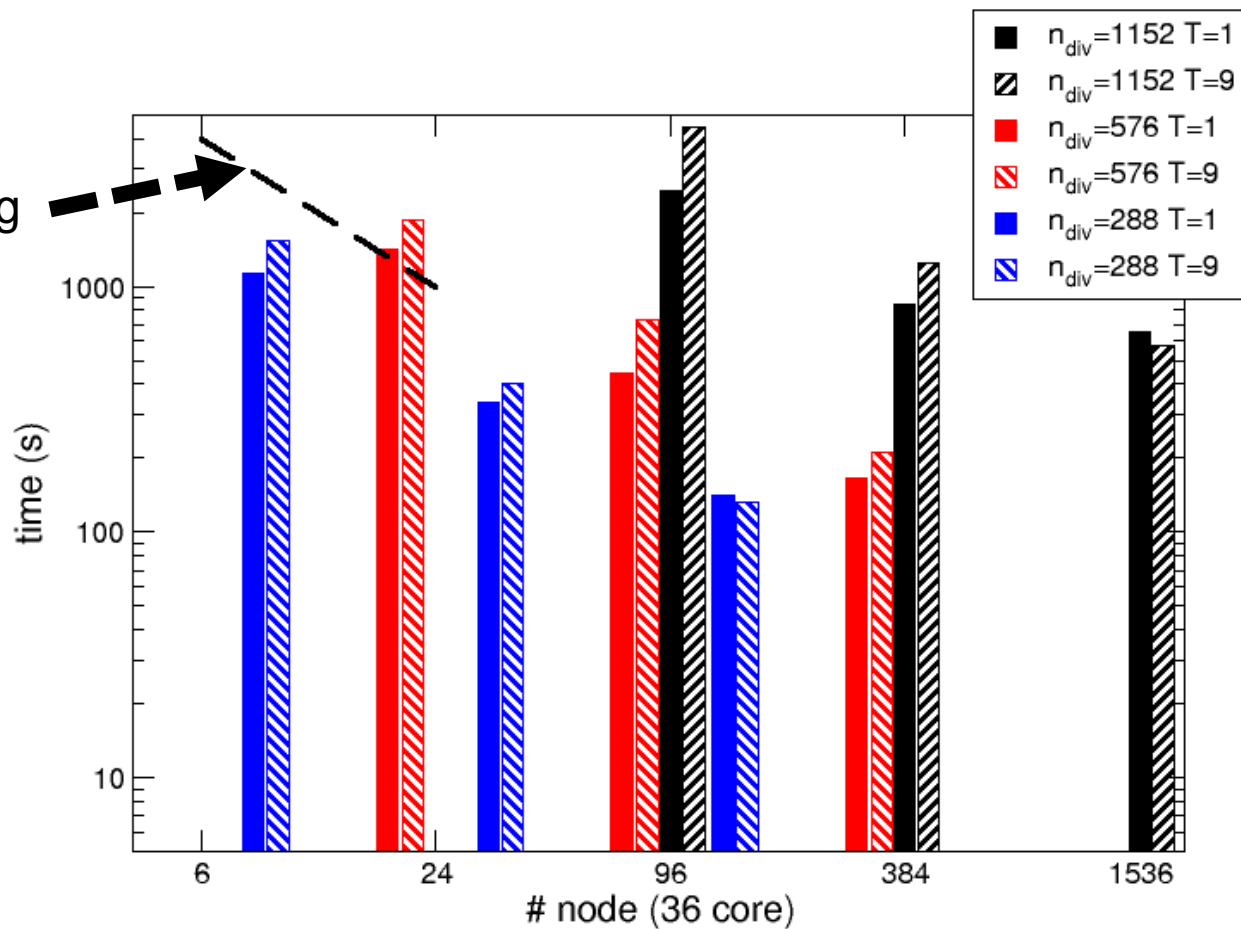


# Where are we?

- Cubed-sphere is principal contender
- But grid non-orthogonal
- To maintain same accuracy using mixed finite-element spatial discretization...
- ...coupled with an ***Eulerian flux form*** transport scheme (either finite element or finite volume)
- Redesigning Unified Model
  - F2003
  - Separation of concerns - PSyKAI
- Targeting early 2023



Perfect scaling



C1152L20  
(9km)

C576L20  
(17km)

C288L20  
(35km)

### Intel Broadwell 36 core node

- 1536 nodes=55296 cores (2/3 machine)
- Largest problem is ~8M cells (20L)

Thank you!

Questions?

See extra slides for  
**Bibliography** and **How**  
**to select options in UM**



# Bibliography

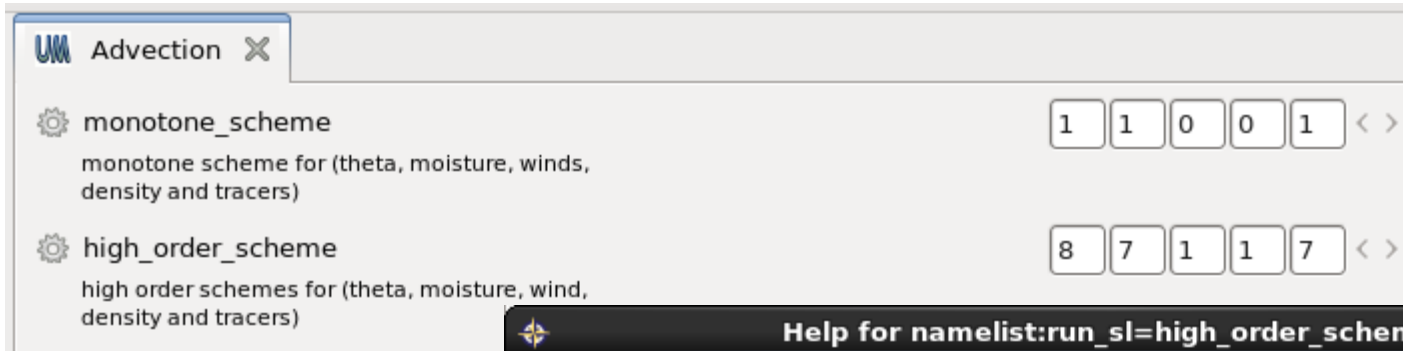
1. Aranami, K., Davies, T. & Wood, N. (2015) , A mass restoration scheme for limited area models with semi-lagrangian advection, *Q. J. R. Meteorol. Soc.* **141**, –. DOI:10.1002/qj.2482.
2. Bermejo, R. & Staniforth, A. (1992) , The conversion of semi-Lagrangian advection schemes to quasi-monotone schemes, *Mon. Wea. Rev.* **120**, 2622–2632.
3. Brown, A., Milton, S., Cullen, M., Golding, B., Mitchell, J. & Shelly, A. (2012) , Unified modeling and prediction of weather and climate: a 25-year journey, *Bull. Amer. Meteor. Soc.* **93**, 1865–1877.
4. Priestley, A. (1993) , A quasi-conservative version of the semi-Lagrangian advection scheme, *Mon. Wea. Rev.* **121**, 621–629.
5. Staniforth, A. & Côté, J. (1991) , Semi-Lagrangian integration schemes for atmospheric models - a review, *Mon. Wea. Rev.* **119**, 2206–2223.
6. Wood, N., Staniforth, A., White, A., Allen, T., Diamantakis, M., Gross, M., Melvin, T., Smith, C., Vosper, S., Zerroukat, M. & Thuburn, J. (2014) , An inherently mass-conserving semi-implicit semi-Lagrangian discretization of the deep-atmosphere global nonhydrostatic equations, *Q.J.R. Meteorol. Soc.* **140**, 1505–1520. DOI:10.1002/qj.2235.
7. Zerroukat, M., Wood, N. & Staniforth, A. (2002) , SLICE: A Semi-Lagrangian Inherently Conserving and Efficient scheme for transport problems, *Q. J. R. Meteorol. Soc.* **128**, 2801–2820.
8. Zerroukat, M. And Shipway, B. (2017) , ZLF (Zero Lateral Flux): A simple mass conservation method for semi-Lagrangian based limited area models, Submitted to *Q. J. R. Meteorol. Soc.*

# Tracer transport options in Rose


with thanks to Chris Smith


# Interpolation options in Rose: $vn \geq 10.6$

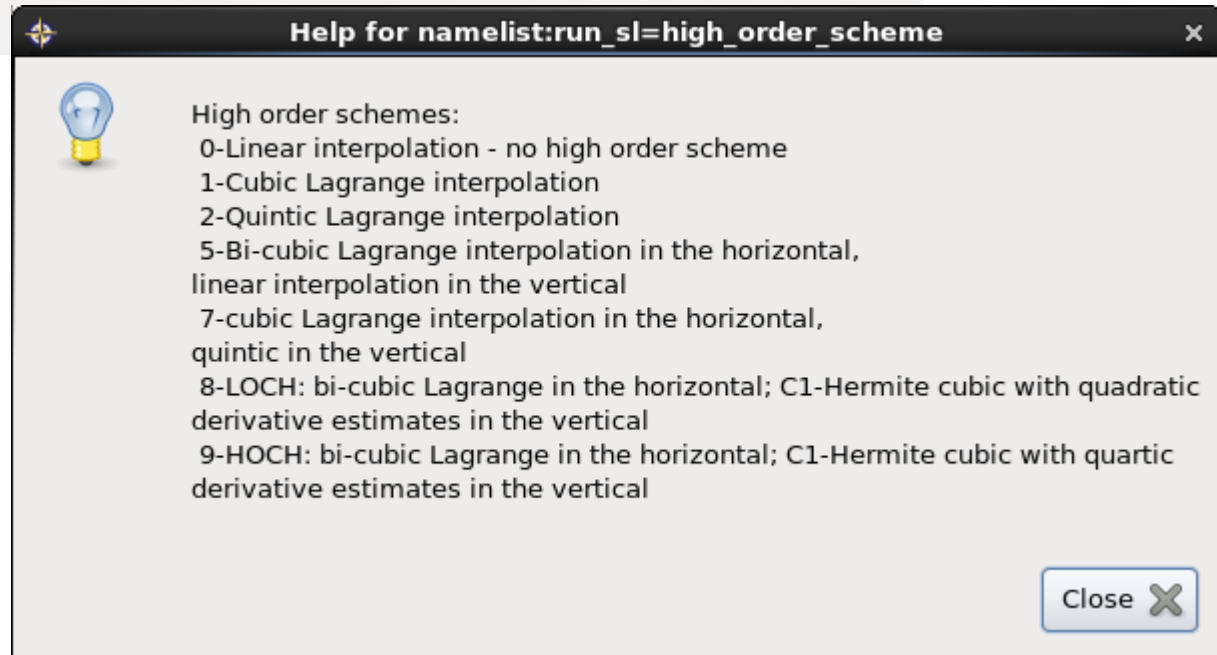
Separate options for moisture and tracers ...




UM Advection X

 **monotone\_scheme** 1 1 0 0 1 < >  
monotone scheme for (theta, moisture, winds,  
density and tracers)

 **high\_order\_scheme** 8 7 1 1 7 < >  
high order schemes for (theta, moisture, wind,  
density and tracers)



**Help for namelist:run\_sl=high\_order\_scheme**

 High order schemes:

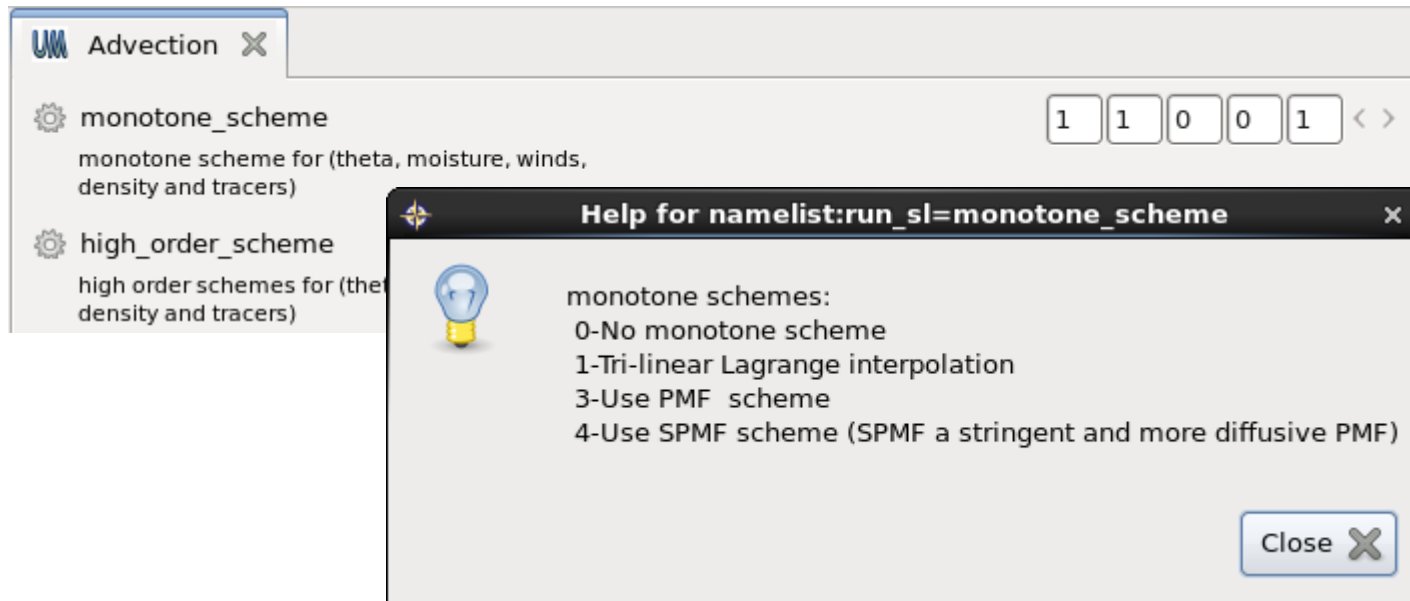
- 0-Linear interpolation - no high order scheme
- 1-Cubic Lagrange interpolation
- 2-Quintic Lagrange interpolation
- 5-Bi-cubic Lagrange interpolation in the horizontal,  
linear interpolation in the vertical
- 7-cubic Lagrange interpolation in the horizontal,  
quintic in the vertical
- 8-LOCH: bi-cubic Lagrange in the horizontal; C1-Hermite cubic with quadratic  
derivative estimates in the vertical
- 9-HOCH: bi-cubic Lagrange in the horizontal; C1-Hermite cubic with quartic  
derivative estimates in the vertical

Close X

... with range of  
interpolation schemes

# Interpolation options in Rose: $vn \geq 10.6$

... and new options for monotonicity





# Conservation options in Rose: $vn \geq 10.1$

Tracer conservation now has the option to use the Priestley (1993) algorithm:

